Lemma 2 の略証

簡単のために, $a(t,\xi) \geq 0$ とする.

$$\kappa(\xi) := \int_0^\delta a(t,\xi)\,dt$$

注）$a(t,0) \not\equiv 0 \Leftrightarrow \kappa(0) \neq 0$

$\Rightarrow$ Weierstrass の予備定理が適用できる

$\kappa(0) = 0$ と仮定する. 広中の resolution thm ([A] 参) より

$\exists U$: open nbd of $0 \in \mathrm{R}^n$, $\exists \widetilde{U}$: real anal. manifold

$\exists \varphi$: $\widetilde{U} \to U$: proper anal. map satisfying the following:

(i) φ: $\widetilde{U}(\xi^0) \setminus \widetilde{A} \to U(\xi^0) \setminus A$: 位相同型

ここで $A = \{\xi \in U; \kappa(\xi) = 0\}$, $\widetilde{A} = \varphi^{-1}(A)$

(ii) $\forall p \in \widetilde{U}$,

$\exists X\ (\equiv X^p) = (X_1, \cdots, X_n)$: **local coord. centered at** p,

$\exists r(p) \in \mathrm{Z}_+$ **with** $r(p) \leq n$, $\exists s_k(p) \in \mathrm{N}$ **(** $1 \leq k \leq r(p)$**)**,

$\exists \widetilde{U}(p)$: **nbd of** p, $\exists e(X)$: **real anal. fun. in** $\widetilde{V}(p)$

s.t. $e(X) > 0$ **for** $X \in \widetilde{V}(p)$,

$$\kappa(\varphi(\tilde{u})) = e(X(\tilde{u})) \prod_{k=1}^{r(p)} X_k(\tilde{u})^{2s_k(p)} \quad (\ \tilde{u} \in \widetilde{U}(p))$$

ここで $\widetilde{V}(p) = \{X(\tilde{u});\ \tilde{u} \in \widetilde{U}(p)\}$,

$r(p) = 0$ のとき $\prod_{k=1}^{r(p)} \cdots = 1$

$\tilde{\varphi}(\equiv \tilde{\varphi}_p)$: $\widetilde{V}(p) \ni X \mapsto \tilde{\varphi}(X) \in U$ を

$$\tilde{\varphi}(X(\tilde{u}))\,(\equiv \tilde{\varphi}_p(X^p(\tilde{u}))) = \varphi(\tilde{u}) \quad \text{for } \tilde{u} \in \widetilde{U}(p)$$

$$U_0\,(\subset U)\text{: compact nbd of } 0, \qquad \widetilde{U}_0 := \varphi^{-1}(U_0)$$

とおく. $p \in \widetilde{U}_0$: 固定

$$\alpha(p) := (s_1(p), \cdots, s_{r(p)}(p), 0, \cdots, 0) \in (\mathbf{Z}_+)^n$$

とし, X で Taylor 展開する:

$$a(t, \tilde{\varphi}(X)) = \sum_{\alpha} c_\alpha(t;p) X^\alpha, \qquad c_\alpha(t;p) = \frac{1}{\alpha!} \partial_X^\alpha a(t, \tilde{\varphi}(X))|_{X=0}$$

$$S_p = \{\alpha \in (\mathrm{Z}_+)^n;\ \ c_\alpha(t;p) \not\equiv 0 \text{ in } t\}.$$

とおく．そのとき $\nu = (\nu_, \cdots, \nu_n) \in (\mathrm{Z}_+)^n$ に対して

$$\int_0^\delta a(t, \tilde{\varphi}(X))\, dt|_{X_k = s^{\nu_k}\ (1 \le k \le n)} \approx s^{2\alpha(p)\cdot\nu} \quad \text{as } s \downarrow 0$$

これより $2\alpha(p) \in S_p$ で，$\alpha \geq 2\alpha(p)$ for $\alpha \in S_p$ を示せる．($\leftarrow$

<u>Lemma</u> $S \subset (\mathrm{Z}_+)^n$, $\beta^0 \in S$ とし，$\exists \beta^1 \in S$ s.t. $\beta^0 \not\leq \beta^1$ とする．そのとき $\exists \nu^0 \in (\mathrm{Z}_+)^n$, $\exists \alpha^0 \in S$ s.t. $\alpha^0 \neq \beta^0$ and

$$\alpha^0 \cdot \nu^0 < \alpha \cdot \nu^0 \quad \text{for } \alpha \in S \setminus \{\alpha^0\}$$

$\leftarrow$ n についての帰納法で示せる([W9] 参))　故に

$$a(t, \tilde{\varphi}(X)) = X^{2\alpha(p)}(c_{2\alpha(p)}(t;p) + b(t, X; p)), \qquad b(t, 0; p) = 0$$

とかけ

$$a(t, X; p) := c_{2\alpha(p)}(t; p) + b(t, X; p)$$

に $(t, X) = (0, 0)$ で **Weierstrass** の予備定理が適用でき，

$$\begin{aligned} &a(t, X; p) \\ &= c(t, X; p)(t^{m(p)} + a_1(X; p)t^{m(p)-1} + \cdots + a_{m(p)}(X; p)) \end{aligned}$$

とかける．後は **compactness argument** を用いて **Lemma 2** を示せる．